

**Number Theory**

Place all answers in the blank space provided. Calculators are permitted.

**You are not expected to answer all the questions.**

- \_\_\_\_\_ 1. A sequence of numbers is defined as follows:  $s_1 = 1$ ,  $s_2 = 1 + 3$ ,  $s_3 = 1 + 3 + 5$ ,  
 $s_4 = 1 + 3 + 5 + 7$ ,  $s_5 = 1 + 3 + 5 + 7 + 9$ , ... . Find  $s_{10}$ .
- \_\_\_\_\_ 2. Number 1 is both a cube and a fourth power. What is the next integer that is both  
a cube and a fourth power?
- \_\_\_\_\_ 3. The symbol  $\tau(n)$  denotes the number of positive divisors of  $n$ . Calculate  $\tau(2012)$ .
- \_\_\_\_\_ 4. The symbol  $\sigma(n)$  denotes the sum of positive divisors of  $n$ . Find  $\sigma(2012)$ .
- \_\_\_\_\_ 5. Lucas numbers are defined recursively:  $L_{n+1} = L_n + L_{n-1}$ , with the initial values  
 $L_0 = 2$  and  $L_1 = 1$ . Find  $L_{17}$ .
- \_\_\_\_\_ 6. What are the largest twin primes both smaller than 102?
- \_\_\_\_\_ 7. Represent 41712 by the use of the Fundamental Theorem of Arithmetic.
- \_\_\_\_\_ 8. Does the equation  $a^{2012} + b^{2012} = c^{2012}$  have a solution in positive integers?
- \_\_\_\_\_ 9. Solve for  $x$  in the interval of least positive residues:  $4x + 17 \equiv 2012 \pmod{25}$ .
- \_\_\_\_\_ 10. Solve for  $x$  in the interval of least positive residues:  $x^2 \equiv -1 \pmod{29}$ .
- \_\_\_\_\_ 11. Calculate the  $\gcd(1704, 2012)$ .
- \_\_\_\_\_ 12. What is the smallest positive integer  $n$  with the following property:  
when divided by 2, the remainder is 1, when divided by 3, the remainder is 2,  
when divided by 4, the remainder is 3, when divided by 5, the remainder is 4,  
when divided by 6, the remainder is 5, when divided by 7, the remainder is 6,  
when divided by 8, the remainder is 7, when divided by 9, the remainder is 8,  
and when divided by 10, the remainder is 9.
- \_\_\_\_\_ 13. Express 2012 as a number in base 4?
- \_\_\_\_\_ 14. Convert  $41712_8$  into base 10.
- \_\_\_\_\_ 15. Find a primitive Pythagorean triple with 17 as one of the numbers.
- \_\_\_\_\_ 16. Does the equation  $987x + 654y = 3210$  have a solution in integers?