**2012 MATH FIELD DAY** **NAME**____________

**SCHOOL**____________

## Logic and Sets

Symbols

| | |
|---|---|
| " $\wedge$ " is the logical connective "and" | " $\Leftrightarrow$ " means "if and only if" |
| " $\vee$ " is the logical connective "or" | " $\equiv$ " means logical equivalence |
| "$\neg p$" is the negation of the statement $p$ | "$\forall$" is the universal quantifier |
| " $\Rightarrow$ " means implication | "$\exists$" is the existential quantifier |
| " $\in$ " means "belongs to" | "$\mathbb{R}$" means the set of real numbers |

**I. Logic. True or False or Unknown: (circle only one in each problem)**

1. T F U If $x \in A$ and $x \in B$, then $x \in (A \cap B)$.

2. T F U 2+3=5 $\Leftrightarrow$ 1+1=3.

3. T F U If $p$ is False, then $p \Rightarrow q$ is True.

4. T F U If $p \vee q$ is True, then what is the status of $p$?

5. T F U Suppose that $p$ is True, $q$ is True, and $r$ is Unknown. What is the status of $(p \vee q) \vee (q \wedge r)$?

6. T F U Suppose that $p$ is True, $q$ is True, and $r$ is Unknown. What is the status of $(p \wedge q) \Rightarrow (q \vee r)$?

7. T F U The negation of $(\exists x, x \geq 2012)$ is $(\forall x, x \leq 2012)$.

8. T F U The inverse of $p \Rightarrow \neg q$ is $q \Rightarrow p$.

9. T F U A contradiction can sometimes be true.

10. T F U The negation of "If Sally doesn't want to be mean, then she must have gotten up on the wrong side of the bed." is "Sally wants to be mean and she must have gotten up on the right side of the bed."

11. T F U $(\exists y \in \mathbb{R})(\exists x \in \mathbb{R})\ \big((x < y) \Rightarrow (x^2 < y^2)\big)$.

12. T F U $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R})\ \big((x < y) \Rightarrow (x^2 < y^2)\big)$.

13. T F U $(\neg p \Rightarrow p) \Rightarrow \neg p$.

14. T F U A contradiction is always true.

15. T F U The complete truth table for $(p \vee q) \wedge (p \vee r) \vee (s \vee t)$ requires 32 lines.

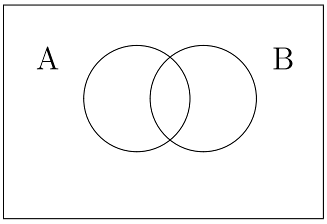
16. T F U $p \vee (q \wedge r) \equiv (p \wedge q) \vee (p \wedge r)$.

Symbols

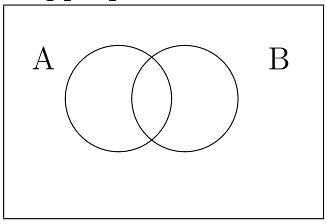
| | | |
|---|---|---|
| $\mathbb{N} = \{0, 1, 2, 3, ...\}$ is the set of natural numbers | $U$ is the universal set | $\emptyset$ is the empty set |
| $\overline{S}$ is the complement of $S$ | The set $A - B$ is the set $A \cap \overline{B}$ | $\subseteq$ means 'is a subset of' |
| $n(S)$ denotes the number of elements in $S$ | $\in$ means 'is an element of' | $\subset$ means 'is a proper subset of' |
| $\sim$ means 'is equivalent to' | $A \times B$ denotes the Cartesian Product | $\mathcal{P}(A)$ is the power set of a set A |

**II. Sets**

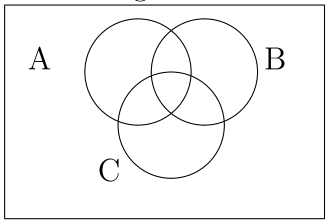
**Problems 1-3:** Shade the appropriate area for each Venn Diagram.




1. $A \cup \overline{B}$




2. $U - (\overline{A} \cap \overline{B})$




3. $(A \cup C) \cup (B \cap \overline{C})$

_____ 4. If there are 10 blue balls, 6 red balls, and 5 black balls in a box together, how many balls do I have to pick in order to assure that I have at least three red balls?

_____ 5. How many integers between 1 and 2012 are divisible by 5 or 11?

**For problems 6-13:** Let $A = \{1, 2, 4\}, B = \{5, 6, 9\}, C = \{0, 4, 8\}, D = \{1, 2, 3\}$, $E = \{3, 6\}$, and $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universal set.

Using the sets just described, describe each of the following sets by listing its elements in {}.

_____ 6. $(C \cap A) \cup B$

_____ 7. $\overline{((A \cap \overline{B}) \cup D \cup \overline{E})}$

_____ 8. $(A \cap \overline{B}) \cup E$

_____ 9. $(A \cap E) \cap ((C \cap D) \cup (D \cap E))$

Using the sets described above, determine if the following statements are true or false.

_____ 10. $(U - C) \subseteq (A \cup B \cup D)$

_____ 11. $n(C \times D) = 9$

_____ 12. $n(\mathcal{P}(E)) = 8$

_____ 13. $(1, 4) \in A \times C$

_____ 14. If $\mathbb{N} = U$, then what is $\overline{\{1, 2, 4, 5, 7, 8, 10, 11, ...\}}$?

_____ 15. (True/False) The set of the multiples of five has the same cardinality as the set of integers.