2012 Math Field Day **Calculus (12^{th})** Name: _____
School: _____

Circle the best answer.

1. Compute the limit $\lim\limits_{x\to 2}\dfrac{x^2-4x+4}{x^3+5x^2-14x}$.

   (a) 0 (b) 2 (c) 4 (d) undefined

2. Which of the following functions are continuous at $x=2$?

   $$f(x)=x^2+4x-12,\quad g(x)=\frac{4x^2+4x+1}{2x-4},\quad h(x)=\frac{x-2}{x-15}$$

   (a) $f,\ g$ (b) $f,\ h$ (c) $g,\ h$ (d) $f,\ g$ and h

3. A function is defined by $f(x)=\sqrt{x+5}$, find $\lim_{h\to 0}\dfrac{f(4+h)-f(4)}{h}$.

   (a) $\dfrac{1}{2}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{6}$ (d) undefined

4. If $f(x)=\dfrac{\sqrt{x}}{1+\sqrt{x}}$, find $f'(x)$.

   (a) $\dfrac{1}{2\sqrt{x}\left(\sqrt{x}+1\right)^2}$ (b) $\dfrac{\sqrt{x}}{\left(1+\sqrt{x}\right)^2}$ (c) $\dfrac{1}{2\left(1+\sqrt{x}\right)^2}$ (d) $\dfrac{1-\sqrt{x}}{1+\sqrt{x}}$

5. If $f(1)=5$, $g(1)=-4$, $f'(1)=-2$ and $g'(1)=12$, find $(fg)'(1)$.

   (a) 68 (b) 52 (c) -44 (d) -34

6. On what intervals are $\dfrac{\tan x}{x}$ continuous?

   (a) $(-2\pi,0)\cup(0,2\pi)$ (b) $\left(-\dfrac{3\pi}{2},0\right)\cup\left(0,\dfrac{3\pi}{2}\right)$ (c) $(-\pi,0)\cup(0,\pi)$ (d) $\left(-\dfrac{\pi}{2},0\right)\cup\left(0,\dfrac{\pi}{2}\right)$

7. Find the slope of the tangent to the curve $x^2+2y^2=9$ at the point $(1,2)$.

   (a) -1 (b) $\dfrac{-1}{2}$ (c) $\dfrac{-1}{3}$ (d) $\dfrac{-1}{4}$

8. Find all critical numbers of $f(x)=2x^3-9x^2+12x$.

   (a) $1,\ -2$ (b) $-1,\ 2$ (c) $1,\ 2$ (d) $-1,\ -2$

9. If $f(x)=\csc^3\left(\dfrac{2}{x}\right)$, find $f'(x)$.

   (a) $6\csc^3\left(\dfrac{2}{x}\right)\cot\left(\dfrac{2}{x}\right)$ (b) $\dfrac{6}{x^2}\csc^3\left(\dfrac{2}{x}\right)\cot\left(\dfrac{2}{x}\right)$ (c) $\dfrac{6}{x^4}\csc^3\left(\dfrac{2}{x}\right)\cot\left(\dfrac{2}{x}\right)$ (d) $\dfrac{6}{x^6}\csc^3\left(\dfrac{2}{x}\right)\cot\left(\dfrac{2}{x}\right)$

10. Calculate $\lim_{x\to\infty}\left(x-\sqrt{x^2+x}\right)$.

(a) 0 (b) -1 (c) $\dfrac{-1}{2}$ (d) undefined

11. If $f''(x)>0$ for all x, then it is always true that

(a) f is increasing
(b) f is decreasing
(c) f' is increasing
(d) f' is decreasing

12. If $f(x)=3^x$, find $f'(x)$.

(a) $3^x\ln 3$ (b) $x3^{x-1}$ (c) 3^{x-1} (d) $x3^{x-1}\ln 3$

13. $\int (7x-2)^3\,dx$ equals

(a) $\dfrac{(7x-2)^4}{4}+C$ (b) $\dfrac{(7x-2)^4}{7}+C$ (c) $\dfrac{7(7x-2)^4}{4}+C$ (d) $\dfrac{(7x-2)^4}{28}+C$

14. Compute $\int_1^2 \dfrac{3x^2+14x}{\left(x^3+7x^2\right)^2}dx$.

(a) $\dfrac{7}{72}$ (b) $\dfrac{5}{72}$ (c) $\dfrac{1}{72}$ (d) None of these

15. Calculate $\int x^7\left(x^4-10\right)dx$.

(a) $\dfrac{1}{16}x^8\left(x^4-10\right)^2+C$ (b) $\dfrac{1}{8}x^8\left(x^5-10\right)^2+C$ (c) $\dfrac{1}{12}x^{12}-\dfrac{5}{4}x^8+C$ (d) $\dfrac{1}{12}x^{12}-\dfrac{5}{3}x^6+C$

16. Calculate $\int_0^{\pi/2}\sin^5\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)dx$.

(a) $\dfrac{1}{128}$ (b) $\dfrac{1}{64}$ (c) $\dfrac{1}{16}$ (d) $\dfrac{1}{4}$

17. Find the area of the region bounded by the curves $f(x)=2x^2$ and $g(x)=x^4-2x^2$.

(a) $\dfrac{128}{5}$ (b) $\dfrac{128}{15}$ (c) $\dfrac{128}{75}$ (d) None of these

18. Find the volume of the solid generated by revolving the curve $y = x^2 + 2$ about the x-axis between $0 \leq x \leq 2$.

(a) $\frac{376}{15}\pi$ (b) $\frac{752}{15}\pi$ (c) $\frac{376}{5}\pi$ (d) $\frac{752}{5}\pi$

19. Suppose thta the edge lengths x, y, and z of a closed rectangular box are changing at the rates

$$\frac{dx}{dt} = 1m/\sec, \frac{dy}{dt} = -\frac{1}{2}m/\sec, \frac{dz}{dt} = 2m/\sec.$$

Find the rates of change of the diagonal when $x = 4$, $y = 3$, $z = 2$.

(a) $\frac{13\sqrt{29}}{58}$ (b) $\frac{8\sqrt{29}}{3}$ (c) $\frac{\sqrt{29}}{115}$ (d) None of these

20. Suppose that $f(x)$, $g(x)$ and $h(x)$ are continuous functions and satisfy $f(x) \leq g(x) \leq h(x)$ for $-2 < x < 2$, then which one of the following must be true?

(a) $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = \lim_{x\to 0} h(x)$

(b) $\lim_{x\to 1} f(x) = \lim_{x\to 1} g(x) = \lim_{x\to 1} h(x)$

(c) $\lim_{x\to 2} f(x) = \lim_{x\to 2} g(x) = \lim_{x\to 2} h(x)$

(d) None of above