2016 MATH FIELD DAY

NAME____________

SCHOOL____________

Logic and Sets

Symbols

" $\wedge$ " is the logical connective "and"	" $\Leftrightarrow$ " means "if and only if"
" $\vee$ " is the logical connective "or"	" $\equiv$ " means logical equivalence
"$\neg p$" is the negation of the statement p	"$\forall$" is the universal quantifier
" $\Rightarrow$ " means implication	"$\exists$" is the existential quantifier
" $\in$ " means "belongs to"	"$\mathbb{R}$" means the set of real numbers

I. Logic. True or False or Unknown: (circle only one in each problem)

1. T F U If $x \in A$ and $x \in B$, then $x \in (A \oplus B)$ (the symmetric difference of the two sets).

2. T F U $238^2 + 2016^2 = 2030^2 \Rightarrow (a+b)^2 = a^2 + b^2$.

3. T F U If $p \vee q$ is True, then $p \Rightarrow q$ is ....

4. T F U If $p \Leftrightarrow q$ is True, then what is the status of p?

5. T F U Suppose that p is False, q is True, and r is Unknown. What is the status of $(p \wedge q) \vee (q \wedge r)$?

6. T F U Suppose that p is False, q is Unknown, and r is False. What is the status of $(p \Leftarrow q) \Leftrightarrow (q \wedge r)$?

7. T F U The negation of $(x \geq 2016 \Leftrightarrow f(x) \geq 36)$ is $(x < 2016 \Leftrightarrow f(x) < 36)$.

8. T F U The converse of $\neg p \Leftrightarrow q$ is $q \Leftrightarrow \neg p$.

9. T F U A tautology is always false.

10. T F U The inverse of "If it is raining and sunny outside, then I am in Florida." is "If I am in Florida, then it is raining and sunny outside."

11. T F U $(\forall y \in \mathbb{R})(\forall x \in \mathbb{R})\ \big((x > y) \Rightarrow (x + y \geq 0)\big)$.

12. T F U $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})\ \big((x < y) \Rightarrow (x^3 < x + y - 1)\big)$.

13. T F U $p \Leftarrow (q \wedge r) \equiv (p \Leftarrow q) \wedge (p \Leftarrow r)$.

14. T F U If we take the contrapositive of a contradiction, we get a tautology.

15. T F U There is exactly one False entry in the the column of the truth table for $(p \wedge q) \vee r$.

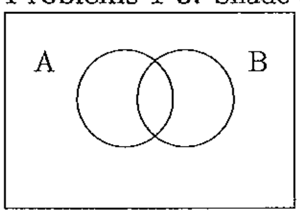
16. T F U $(p \Rightarrow q) \wedge r \equiv (p \wedge r) \Rightarrow (q \wedge r)$.

Symbols

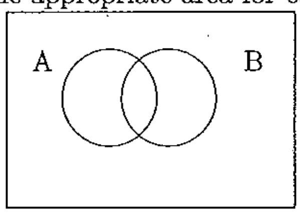
$\mathbb{N} = \{0, 1, 2, 3, ...\}$ is the set of natural numbers	U is the universal set	$\emptyset$ is the empty set
$\overline{S}$ is the complement of S	The set $A - B$ is the set $A \cap \overline{B}$	$\subseteq$ means 'is a subset of'
$n(S)$ denotes the number of elements in S	$\in$ means 'is an element of'	$\subset$ means 'is a proper subset of'
$\sim$ means 'is equivalent to'	$A \times B$ denotes the Cartesian Product	$\mathcal{P}(A)$ is the power set of a set A

II. Sets

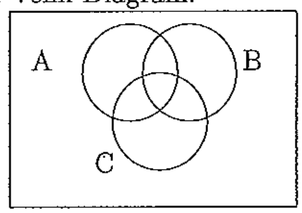
Problems 1-3: Shade the appropriate area for each Venn Diagram.



1. $(A \cap B) \cup (\overline{A} \cap B)$



2. $A - (U - B)$



3. $(A \cap B) \cup (A \cap \overline{C})$

_____ 4. If there are 20 blue balls, 16 red ball, and 2016 black balls in a box together, how many balls do I have to pick in order to assure that I have at least five red balls and twenty black balls?

_____ 5. How many integers between 1 and 2016 are divisible by both 19 and 23?

For problems 6-13: Let $A = \{1, 2, 8\}, B = \{5, 6, 7\}, C = \{0, 1, 2, 4, 8\}, D = \{1, 2, 5\}$, $E = \{0, 5\}$, and $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universal set.

Using the sets just described, describe each of the following sets by listing its elements in { }.

_____ 6. $(B - \overline{A}) \cup C$

_____ 7. $(A - \overline{B}) \cap ((A - C) \cup E)$

_____ 8. $(A \cup \overline{B}) \cap E$

_____ 9. $(A \cap E) \cap ((B \cup E) \cup (D \cap B))$

Using the sets described above, determine if the following statements are true or false.

_____ 10. $n(C - A) = n(E)$

_____ 11. $n(C \times E) = 7$

_____ 12. $n(\mathcal{P}(E)) = 4$

_____ 13. $\emptyset \subseteq C \times E$

_____ 14. (True/False) The set of transcendental numbers is countable.

_____ 15. (True/False) The empty set is an element of every power set.