

**Number Theory**

Place all answers in the blank space provided. Calculators are permitted.

**You are not expected to answer all the questions.**

- \_\_\_\_\_ 1. Represent 1442016 by the use of the Fundamental Theorem of Arithmetic.
- \_\_\_\_\_ 2. The symbol  $\tau(n)$  denotes the number of positive divisors of  $n$ . Calculate  $\tau(2016)$ .
- \_\_\_\_\_ 3. The symbol  $\sigma(n)$  denotes the sum of positive divisors of  $n$ . Find  $\sigma(2016)$ .
- \_\_\_\_\_ 4. The symbol  $\varphi(n)$  denotes the number of positive integers not greater than  $n$  that are relatively prime to  $n$ . Find  $\varphi(2016)$ .
- \_\_\_\_\_ 5. Triangular numbers are defined as follows:  $t_1 = 1$ ,  $t_n = t_{n-1} + n$  for  $n > 1$ . Find  $t_{2016}$ .
- \_\_\_\_\_ 6. A perfect number is a positive integer that is equal to the sum of its proper positive divisors. For example  $6 = 1 + 2 + 3$ ,  $28 = 1 + 2 + 4 + 7 + 14$ . What is the next smallest perfect number?
- \_\_\_\_\_ 7. Give an example of a positive integer  $n$  such that  $\varphi(n)$  is odd.
- \_\_\_\_\_ 8. What are the largest twin primes both smaller than 1000?
- \_\_\_\_\_ 9. Does the equation  $a^{2016} - b^{2016} = c^{2016}$  have a solution in negative integers?
- \_\_\_\_\_ 10. Does the following congruence  $14x \equiv 4 \pmod{2016}$  have a solution?
- \_\_\_\_\_ 11. The number  $n = 4 \cdot 10^{2016} + 14$  is even, so divisible it is by 2. What is the next smallest prime divisor of  $n$ ?
- \_\_\_\_\_ 12. Find the remainder when  $414^{2016}$  is divided by 7.
- \_\_\_\_\_ 13. Express 2016 as a hexadecimal number.
- \_\_\_\_\_ 14. Convert  $(2016)_8$  into base 10.
- \_\_\_\_\_ 15. Find a Pythagorean triple with 14 as one of the numbers.
- \_\_\_\_\_ 16. The  $\gcd(414, 2016) = 18$ . Find a pair integers  $(x, y)$  such that  $414x + 2016y = 18$ .
- \_\_\_\_\_ 17. For which of the following prime numbers  $p$ , does the congruence  $x^2 \equiv -1 \pmod{p}$  have a solution? Choices for  $p$ : 419, 2017, 5021, 9967.