

Symbols You are not expected to answer all the questions.

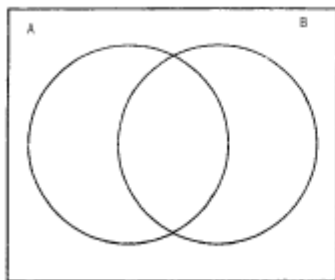
“ \wedge ” is the logical connective “and”.	“ \forall ” is the universal quantifier.	U is the universal set.
“ \vee ” is the logical connective “or”.	“ \exists ” is the existential quantifier.	The set $A - B$ is the set $A \cap \bar{B}$.
“ $\neg p$ ” is the negation of the statement p .	$\mathbb{N} = \{0, 1, 2, 3, \dots\}$ natural numbers. \mathbb{Z} is the set of integers.	\in means ‘is an element of’.
“ \Rightarrow ” means implication.	\mathbb{Q} is the set of rational numbers. \mathbb{R} is the set of real numbers.	$A \times B$ denotes the Cartesian product of sets A and B .
“ \in ” means “belongs to”	\bar{S} is the complement of S .	\emptyset is the empty set.
“ \Leftrightarrow ” means “if and only if”.	$n(S)$ denotes the number of elements in S .	\subseteq means ‘is a subset of’. \subset means ‘is a proper subset of’.
“ \equiv ” means logical equivalence.	\sim means ‘has the same cardinality as’	$\mathcal{P}(A)$ is the power set of a set A .

I. Logic True or False or Unknown: (circle only one in each problem)

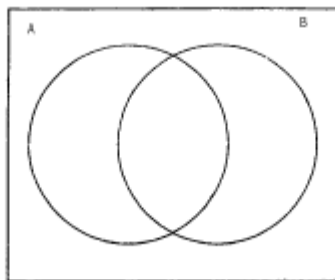
1. T F U $\forall x \in \mathbb{N}, \forall y \in \mathbb{Z}, x^2 > y^2 \Rightarrow x > y$
2. T F U If $p \Rightarrow q$ is True, then what is the status of $p \wedge q$?
3. T F U If $p \Leftrightarrow q$ is False, then what is the status of $(\neg p \vee q) \Rightarrow (p \wedge q)$?
4. T F U If p and q are False and r is Unknown, then $(p \Rightarrow q) \Rightarrow r$ is Unknown.
5. T F U The negation of “At least five students passed the test” is “At most five students failed the test”.
6. T F U $\exists a \in \mathbb{Z}, \exists b \in \mathbb{Z}, ab = 2023$
7. T F U $\exists a \in \mathbb{R}, \forall b \in \mathbb{R}, ab = 2023$
8. T F U $(\forall p \exists q, p \Rightarrow q) \equiv \neg(\exists p \forall q, q \Rightarrow p)$
9. T F U $((p \vee q) \wedge (q \vee p)) \equiv (p \wedge q)$?
10. _____ P and Q are tautologies. Use both of them and the symbols \Rightarrow and \neg (once each) to construct a contradiction.
11. _____ How many rows does the truth table for $p_1 \vee p_2 \vee \dots \vee p_7$ have?

II. Sets

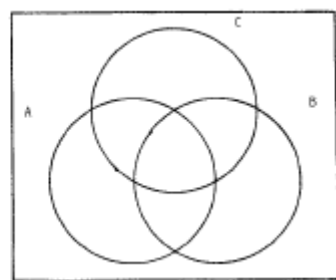
Problems 1-3: Shade the appropriate area for each Venn Diagram.



1. $(\bar{A} \cup \bar{B}) \cap \overline{(A - B)}$



2. $\overline{((A \cup B) - (B \cap A))}$



3. $A \cap B \cap \bar{C}$

For problems 4-11: Let $A = \{2,3,5,7\}$, $B = \{0,2,4,6,8\}$, $C = \{0,1\}$, $D = \{0,1,3,6,9\}$, $E = \{0,1,8\}$, and $U = \{0,1,2,3,4,5,6,7,8,9\}$ the universal set.

Using the sets just described, describe each of the following sets by listing its elements in $\{ \}$

_____ 4. $\bar{D} \cap \overline{(A \cup B)}$

_____ 5. $B - \bar{C}$

_____ 6. $B \cap (C \cup E)$

_____ 7. $(A - D) - D$

_____ 8. $(B \cap D) \times \overline{(B \cup D)}$

_____ 9. $\mathcal{P}(\bar{U})$

Using the sets described above, determine the answers.

_____ 10. (T/F) $A \cap B \subseteq D$

_____ 11. (T/F) $(A \cup C) \sim (D \cup E)$

_____ 12. $n(\mathcal{P}(\mathcal{P}(E))) = ?$

_____ 13. (T/F) $\mathbb{N} \times \mathbb{N} \sim \mathbb{Z}$

_____ 14. (T/F) For some set S , $A \sim \mathcal{P}(S)$.

_____ 15. (T/F) For every set S , $n(\mathcal{P}(S))$ is even.