

Problem Solving Event: Math Field Day, April 20, 2023

Southeast Missouri State University: Department of Mathematics

Math Field Day 2023 School _____.

Problem Solving Event Sponsor _____.

RETURN THIS FORM SIGNED TO THE REGISTRATION TABLE BEFORE 11:30 A.M.

Note:

1. This set of problems will be given to each school at 9:00 a.m.
2. The problems are each on a separate page attached hereto. The pages may be separated at the direction of the sponsor for the convenience of the students.
3. The sponsor is responsible for the distribution of the problems to his/her students, the collection of the solutions (one solution per problem written on the original problem sheet), and the return of the set of answers by 11:30 a.m. to the Information Table outside Ballroom B in the University Center.
4. The sponsor may not contribute to the solution of any of the problems.

To the best of my knowledge, our school has complied fully with the above procedure.

Sponsor's Signature: _____.

Question 1

The positive integers from 1 to n inclusive are written on a blackboard. After one number is erased, the average (arithmetic mean) of the remaining $n - 1$ numbers is $46\frac{20}{23}$. Determine n and the number that was erased.

Question 2

Two circles, one with radius 1, the other with radius 2, intersect so that the larger circle passes through the center of the smaller circle. Find the distance between the two points at which the circles intersect.

Question 3

Consider the ten numbers ar, ar^2, \dots, ar^{10} . If their sum is 18 and the sum of their reciprocals is 6, determine their product.

Question 4

- a) Determine the two values of x such that $x^2 - 4x - 12 = 0$.
- b) Determine the *one* value of x such that $x - \sqrt{4x + 12} = 0$. Justify your answer.
- c) Determine all real values of c such that:

$$x^2 - 4x - c - \sqrt{8x^2 - 32x - 8c} = 0$$

has precisely two distinct real solutions for x .

Question 5

The points $(2, 5)$ and $(6, 5)$ are two of the vertices of a regular hexagon of side length two on a coordinate plane. There is a line L that goes through the point $(0, 0)$ and cuts the hexagon into two pieces of equal area. What is the slope of line L ?

Question 6

Given any triangle ABC where b and c are the sides opposite to angles B and C , respectively, and AD is a median of length m . Prove that $4m^2 = b^2 + c^2 + 2bc \cos A$.

Question 7

Given that the perimeters of an equilateral triangle T and a square S are equal, determine the ratio of the area of the equilateral triangle T to the area of the square S .

Question 8

The zeroes of the polynomial $f(x) = x^2 - ax + 2a$ are integers. What is the sum of all the possible values of the number a ?

Question 9

In still water Aoife swims at 2 kph. She is standing on the bank of a river that is exactly 100 m wide. The river is flowing past at a speed of 1.2 kph. How long (in seconds) will it take her to swim in a straight line to the point directly across from her on the other bank of the river.

Question 10

What is the value of the positive integer n for which the least common multiple of 36 and n is 500 greater than the greatest common divisor of 36 and n ?