

2023 MATH FIELD DAY : NUMBER THEORY

Name: _____

School: _____

Place all answers in the blank space provided. Calculators are permitted. **You are not expected to answer all the questions.**

1. _____ Represent 2023 by the use of the Fundamental Theorem of Arithmetic.
2. _____ Number 1 is both a cube and a fourth power. What is the next integer that is both a cube and a fourth power?
3. _____ The symbol $\tau(n)$ denotes the number of positive divisors of n . Calculate $\tau(2023)$.
4. _____ The symbol $\sigma(n)$ denotes the sum of positive divisors of n . Find $\sigma(2023)$.
5. _____ Find the least abundant number.
6. _____ The Lucas numbers are a recursively defined sequence with $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$. Find L_{10} .
7. _____ The quantity 11111 is a repunit number (such as 111) as each digit is 1. Find the prime factors of 11111.
8. _____ Does the equation $a^{2023} + b^{2023} = c^{2023}$ have a solution in positive integers?
9. _____ Solve for x in the interval of least positive residues: $2x + 7 \equiv 2023 \pmod{23}$.
10. _____ Solve for x in the interval of least positive residues $x^2 \equiv 20 \pmod{23}$.
11. _____ Calculate the $\gcd(419094123, 9301842)$.
12. _____ When 713^{4899} is divided by 2023, what is the remainder?
13. _____ Express 2023 in hexadecimal form.
14. _____ Convert $(2023)_{23}$ into base 10.
15. _____ Find a primitive Pythagorean triple with 9 as one of the numbers.
16. _____ Does the Diophantine equation $34x + 119y = 2023$ have a solution in the integers?