2023 MATH FIELD DAY Matrices and Determinants

NAME ______________________________ SCHOOL ______________________________

Place all answers in the blank space provided. You are not expected to answer all questions.

Note: | | will denote a determinant and [ ] will denote a matrix.

______________ Q1. Given $A(B+C) = D, AC = \begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}, B^{-1} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$, and $D^{-1} = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$. Find A.

______________ Q2. Given $A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$ and $\det(A) = -2$. Find $\det(3A)$.

______________ Q3. Given $A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}$ and $\det(A) = -4$. Find $\begin{vmatrix} m & n & o & p \\ i & j & k & l \\ e & f & g & h \\ m & n & o & p \end{vmatrix}$

______________ Q4. For what value(s) of s is $\begin{bmatrix} 3 & s \\ s & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 3 & 3 \end{bmatrix}$ a singular matrix?

______________ Q5. Write the two matrices whose determinants need to be evaluated to solve the following set of equations for x by Cramer's Rule.

$$\begin{aligned} 5x - 7y &= 3 \\ -4x + 6y &= 5 \end{aligned}$$

______________ Q6. Compute the determinant $\begin{vmatrix} -4 & 3 & 0 & 0 \\ 2 & -1 & 0 & 0 \\ 0 & 0 & 8 & 7 \\ 0 & 0 & 6 & 5 \end{vmatrix}$

______________ Q7. If M is a 357×22 matrix, what must the dimension of the matrix N be if both MN and NM are defined?

______________ Q8. Find all values of a such that $\begin{vmatrix} -2-a & 4 \\ -5 & 3+a \end{vmatrix} = 0$

______________ Q9. Let $A = \begin{bmatrix} 2 & r \\ s & t \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$. Solve for $r, s,$ and t given that $AB = BA$ and $AC = CA$.

______________ Q10. Find $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [-4 \;\; -5 \;\; -6]$

______________ Q11. Find $\begin{bmatrix} -1 & -1 \\ 0 & 1 \end{bmatrix}^{2023}$