2023 MATH FIELD DAY **NAME:** _____
Calculus (12^{th} GRADE) **SCHOOL:** _____

All answers must be simplified and exact (i.e., they may contain π, radicals, fractions, terminating or repeating decimals, but they may *NOT* contain decimal approximations). Radicals must be expressed in simplest radical form and fractions must be expressed in lowest terms.

1. _____ If $f(x) = x^{\sin(x)}$, find $f'(x)$.

2. _____ If $f(x) = x^e + e^x$, find $f'(1)$.

3. _____ Determine the tangent line to the graph of $xy = \sin(x + y)$ at the point $(0, \pi)$.

4. _____ Calculate $\lim_{x \to 3} \log_2 \left(\frac{x^2 - 6x + 9}{x^2 - 9} + 8 \right)$.

5. _____ Determine the intervals over which the function $f(x) = x^3 - x^2 + 1$ is increasing.

6. _____ Determine the limit $\lim_{x \to 0^+} \frac{\sin(\sqrt{x})}{\sqrt{x}}$.

7. _____ Calculate $\lim_{x \to 0} \frac{\sqrt[4]{16 + x} - 2}{x}$.

8. _____ Compute the integral $\int_0^{\sqrt{3}} \sqrt{3 - x^2} dx$.

9. _____ Compute the integral $\int \frac{1}{\sqrt{x}(x + 1)} dx$.

10. _____ If $F(x) = \int_{x^2}^{x^3} \tan(t) dt$, determine the derivative $\frac{dF(x)}{dx}$.