2023 MATH FIELD DAY : ALGEBRA II

Name: _____ School: _____

Place all answers in the blank space provided in simplified, exact form. **You are not expected to answer all the questions.**

1. _____ Solve over the set of Real Numbers: $20x^2 - 2x^3 = 22x$.

2. _____ Solve the system of equations: $x^2 + y^2 = 27, x^2 - 2y = 3$

3. _____ Write the standard equation of a circle with center at (4,7) and is tangent to the y-axis.

4. _____ Find the highest point on the curve $y = -\frac{3}{16}x^2 + \frac{3}{8}x + \frac{45}{16}$.

5. _____ Solve: $\sqrt{7x+4} - 5 \geq 7$

6. _____ Solve for x: $\log_3(x+3) - \log_3(2) = \log_3(x-1) - \log_3(7)$

7. _____ Write the first two terms in the expansion of $(2x-2)^{13}$

8. _____ If $f(x) = \sqrt{x}$, then find $\frac{f(x+h) - f(x)}{h}$ with no factor of h in the denominator.

9. _____ Find the domain of $(f \circ g)(x)$ given $f(x) = \frac{2}{2x+3}$ and $g(x) = \frac{x+2}{x-3}$.

10. _____ Write an equation for a parabola with vertex $(-4, 7)$ and going through $(2, 3)$.

11. _____ Find all roots of $\frac{4}{x-1} = \frac{8x^2}{x^2-1} - \frac{x}{x+1}$

12. _____ Find all the zeroes of $f(x) = 2x^4 + 4x^3 - x^2 - 6x - 3$ over the set of complex numbers.

13. _____ Write an equation for the ellipse with the following conditions: major axis 10 units long and parallel to the y-axis, minor axis 4 units long, center at $(-2, 7)$.

14. _____ Find the indicated term of the geometric sequence (assume the sequence starts at a_1): a_{10} where the sequence is $14, 2, 2/7, ...$

15. _____ Evaluate $1296^{\frac{7}{4}}$

16. _____ Solve $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 3 = 0$