Problem Solving Event: Math Field Day, April 10, 2025

Southeast Missouri State University: Department of Mathematics

Math Field Day 2025 School____.

Problem Solving Event Sponsor____.

RETURN THIS FORM SIGNED TO THE REGISTRATION TABLE BEFORE 11:30 A.M.

Note:

1. This set of problems will be given to each school at 9:00 a.m.
2. The problems are each on a separate page attached hereto. The pages may be separated at the direction of the sponsor for the convenience of the students.
3. The sponsor is responsible for the distribution of the problems to his/her students, the collection of the solutions (one solution per problem written on the original problem sheet), and the return of the set of answers by 11:30 a.m. to the Information Table outside Ballroom B in the University Center.
4. The sponsor may not contribute to the solution of any of the problems.

To the best of my knowledge, our school has complied fully with the above procedure.

Sponsor's Signature:____.

Question 1:

Let $P(x) = x^4 + ax^3 + bx^2 + cx + d$.

where $a, b, c,$ and d are constants. If $P(1) = 10, P(2) = 20, P(3) = 30$, compute $\frac{1}{10}(P(12) + P(-8))$.

Question 2:

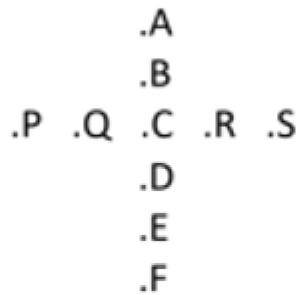
Prove that $2^{2n+5} + 9n^2 + 3n + 4$ is divisible by 18 for any non-negative integer n.

Question 3:

A solid sphere is perfectly embedded in a cube where the side length of the cube is 8 cm. What is the amount of unoccupied space within the cube?

Question 4:

The points A, B, C, D, E, F are collinear and the points P, Q, C, R, S are collinear, as shown.



How many different triangles may be formed by using three of these given points as vertices?

Question 5:

Let $\theta = \arctan 2 + \arctan 3$. Find $\frac{1}{\sin^2 \theta}$ and simplify fully.

Question 6:

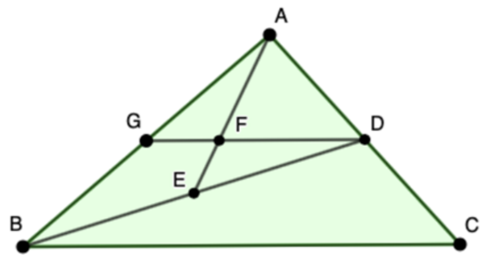
Find the sum of all positive integers x for which $x + 56$ and $x + 113$ are perfect squares.

Question 7:

In the triangle ABC, BD is the median to the side AC, DG is parallel to the base BC (G is the point of intersection of the parallel with AB).

In the triangle ABD, AE is the median to the side BD and F is the intersection point of DG and AE.

Find $\frac{BC}{FG}$.



Question 8:

Consider the ten numbers $ar, ar^2, ar^3, \ldots, ar^{10}$. If their sum is 18 and the sum of their reciprocals is 6, determine their product.

Question 9:

Suppose a sample space consists of 4 elements: $\Omega = \{a, b, c, d\}$. Let P be a probability function defined on Ω with the following:

- $P(\{a\}) = \frac{1}{2}$
- $P(\{a, b\}) = \frac{19}{30}$
- $P(\{a, c\}) = \frac{2}{3}$

Calculate $P(\{b, d\})$.

Question 10:

Each week, an insurance company receives 20 applications for new claims. Of these 20, unknown to the company, 4 are considered fraud. The company selects 3 applications at random, without replacement. Calculate the probability that at most one will be considered fraud (Round your answer to the nearest tenth).